RAW | RISK AT WORK

REPLICATING PORTFOLIOS FOR MORTGAGE PREPAYMENTS

by Bram Jochems and Alexander van Haastrecht

Mortgages are an important asset on the balance sheet for all large Dutch financial institutions. Within mortgages there is embedded interest rate optionality, as mortgagors have the possibilities to prepay part of their mortgage without penalty or break fees each year or prepay in full when moving to another house. Some of the behaviour surrounding exercise is dependent on interest rates and therefore the prepayment option can be seen as interest option at the side of the client. Hence, suitable management of the prepayment behaviour, requires this option to be priced and hedged as embedded interest rate optionality.

Historically, Dutch financial institutions have been applying static prepayment models, that specify a fixed (term structure of) prepayment rate(s) to be applied to the outstanding mortgage portfolio. Doing so underestimates the risks this option poses as it ignores the losses related to the non-linear behaviour of this optionality.

Due to regulatory requirements and advances in computation and modelling capabilities, there is an increasing pressure on financial institutions to use more sophisticated prepayment models.
In this article we describe how the risk management of prepayment optionality can be aided by using replicating portfolios.

The impact of prepayment on mortgage portfolio valuation

Due to their attractive risk-return profile, Dutch banks and insurers have a significant amount of mortgages on their balance sheet. Figure 1 shows outstanding home mortgage lending broken down by sector.

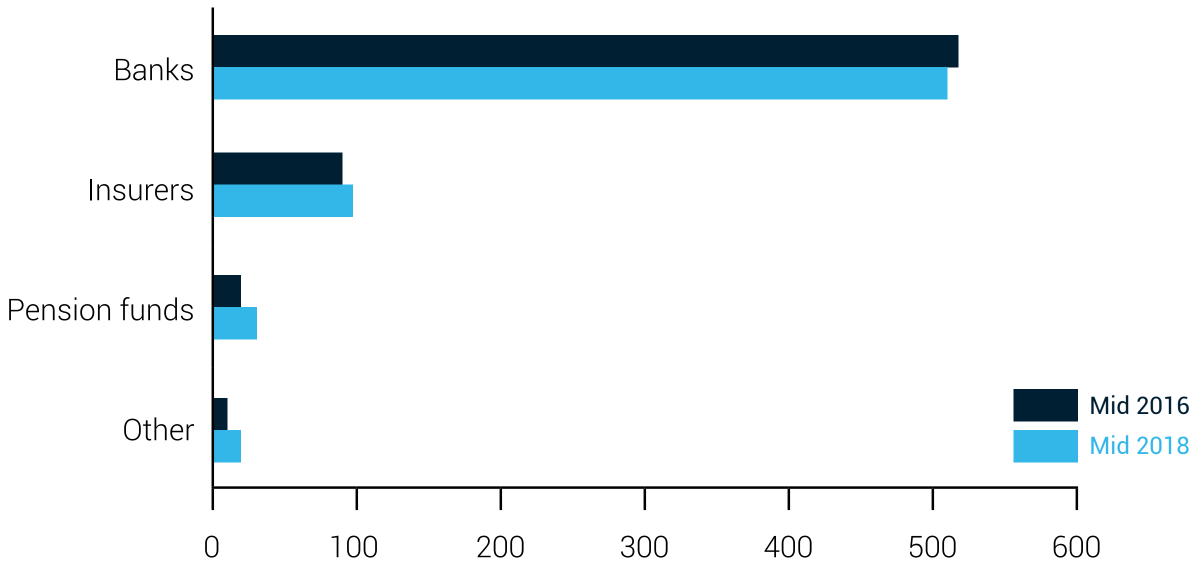

Figure 1: Outstanding home mortgage lending by sector (Source: DNB)

In the Netherlands, typical conditions for retail mortgages allow mortgage holders to prepay penalty free in the case of certain 'life events' (such as moving to another house) and/or to prepay yearly 10%-20% of their original outstanding amount without any penalty payments. For this reason, an important factor for the valuation, risk and capital calculations associated with mortgages, is what prepayment model and/or assumptions are applied.

To illustrate the impact of prepayment rates, figure 2 shows how the present value of two typical mortgages, one without any scheduled notional repayments and one with annuity payments, varies as a function of a static prepayment rate.

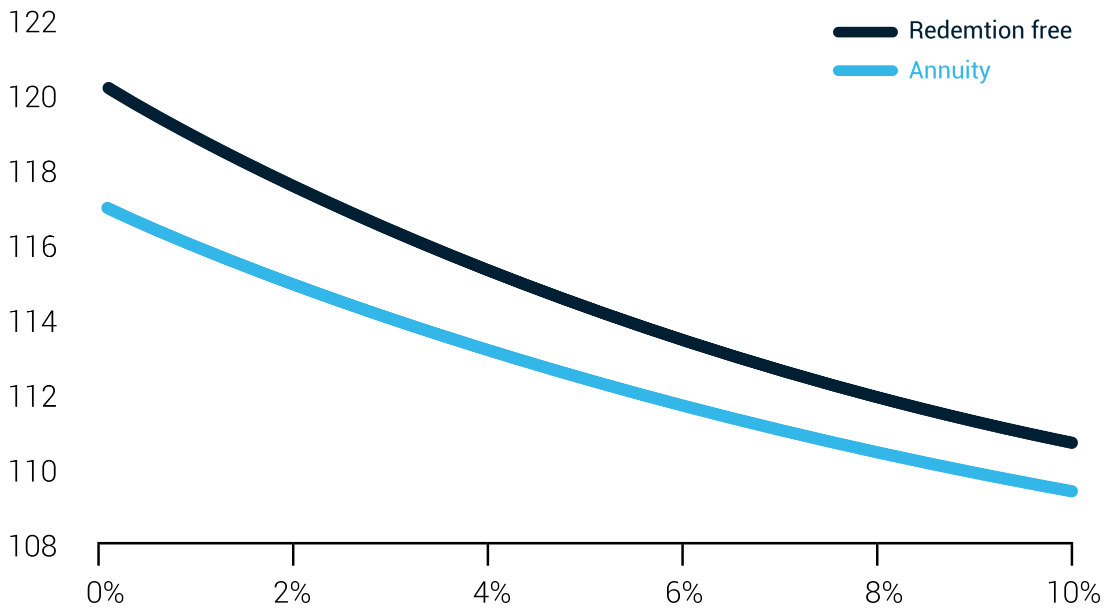

Figure 2: PV of a mortgage as function of a static prepayment rate

Figure 2 established the relationship between the value of a mortgage and a static prepayment rate. Empirical analysis of prepayments shows that prepayments are typically dependent on interest rates; stylistically the relationship is typically similar to figure 3. In this figure a prepayment rate is shown on the y-axis that is a function of refinancing (mortgage) rates on the x-axis. Although the values in the graph will depend on the current and historic economic and interest rate environment, it's typical for prepayment rates to be a non-linear decreasing function of interest (or mortgage) rates. This is easily understood from the observation that as refinancing becomes more attractive, a larger incentive will evoke more actions from mortgagors.

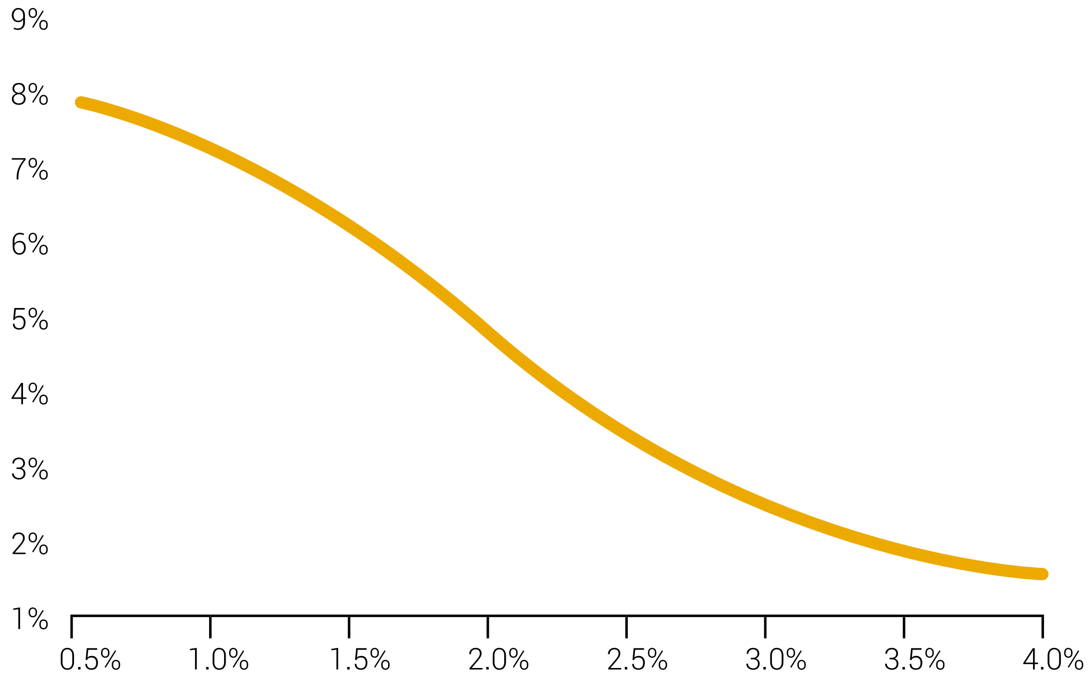

Figure 3: typical behavior for prepayment rate as a function of refinance rates

Together, figure 2 and 3 establish a non-linear relationship between the economic value of a mortgage and interest rates.

Historically, Dutch financial institutions have been applying static prepayment models, that specify a fixed (term structure of) prepayment rate(s) to be applied to the outstanding mortgage portfolio. Due to regulatory requirements and advances in computation and modelling capabilities, there is an increasing pressure on financial institutions to use more sophisticated prepayment models.

> **there is increasing pressure for financial institutions to use more sophisticated prepayment models**

Prepayment option valuation

A prepayment option is a complex, exotic option for various reasons. Amongst those are:

- Sale of property (allowing redemption in full) is dependent on housing market activity, which in turn might be related to interest rates since they affect mortgage affordability.
- Exercise in the case of a sale of a property is quite rational and dependent on mortgage rates;
- Partial prepayments can also be done and can be thought of as a function of (expected future) liquidity availability, incentives vis-a-vis savings rates and expected equity returns and tax regulation;
- There's some evidence that divorce rates (which typically allow for full prepayment without penalties) are dependent on the state of the economy which in turn might be related with interest rates;
- the amount of outstanding notional on which the option can be exercised is path dependent as it matters when people prepay.

For the reasons above, analytical valuation of a prepayment option is not possible. In addition, there are several reasons why monte carlo simulations of the prepayment option are hard to do:

1. Mortgage portfolios on the balance sheets of Dutch financial institutions are typically quite large; even after grouping on common properties, it's not uncommon to see > 1,000,000 mortgages to be valued;

2. Monte Carlo valuation of mortgages in its full generality requires quite expensive simulations as per path, for each mortgage, at each point before exercise, a prepayment rate has to be determined;

3. Client make decisions on their prepayment in the context of their full mortgage coupon; this coupon typically consists of an interest rate, a funding cost and a commercial component and the latter two might not be independent of interest rates.

In practice, one sees that the second and third problem can be worked around by some simplifications, but efficient valuation of large mortgage portfolios remains a problem. Recent advances in computational power, such as pervasive cloud and GPU computing have reduced this problem somewhat, but it still remains very relevant, especially in typical vendor ALM systems.

Replicating portfolios

One solution to the problem of monte carlo valuations being still infeasible for most financial institutions to directly implement, is provided by replicating portfolios. Replicating portfolios are a technique already widely employed by Dutch insurers for other parts of their balance sheets. The concept behind replicating portfolios is to find a portfolio of vanilla financial instruments (e.g. swaps, bonds, or swaptions) that have the same value and risk profile as the underlying portfolio they replicate. If such a portfolio can be found and fits the underlying portfolio sufficiently close, this portfolio can be used for valuation, hedging and/or capital purposes.

Below we will discuss how to find a replicating portfolio, but it will turn out that this still requires monte carlo simulations. However, the problem mentioned earlier on the infeasibility of doing many monte carlo simulations is mitigated with replicating portfolios for two reasons. First, to establish a replicating portfolio, typically one needs less simulations than required to value an option with reasonable accuracy. Secondly, once a replicating portfolio has been found, it can be kept static and revalued analytically very quickly, e.g. on a monthly or quarterly basis.

To find a replicating portfolio users need to specify three input components, i) scenarios, ii) a cash flow model and iii) replicating instruments. Based on these inputs, finding a replicating portfolio can then be formulated as a mathematical optimization problem. This is schematically depicted in figure 4.

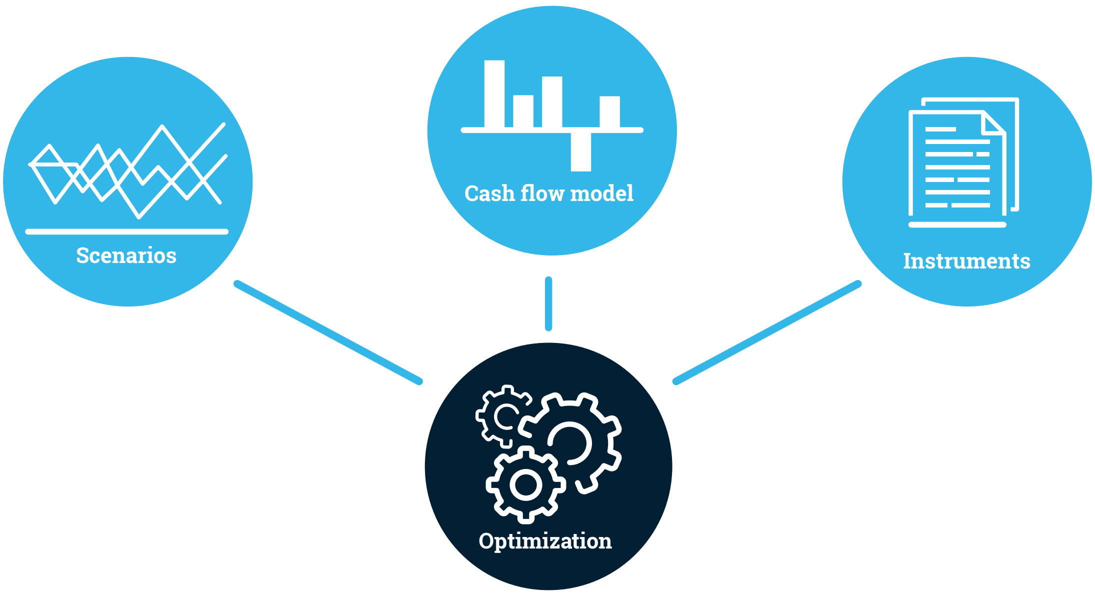

Figure 4: Graphic depiction of Replication methodology

We discuss each component in turn below.

The scenarios that serve as inputs for the optimization problem are stochastic and together generate multiple paths of the evolution of state variables of interest. The goal here is not to find convergence on the value of the underlying product, but rather cover a broad range of the potential search space and potentially focus on the area of interest in the replicating portfolio (e.g. for capital purposes, scenarios representing severe downshocks can be used). In addition, by defining several scenarios, also differences between scenarios (so called sensitivities or greeks) can be matched in the optimization problem.

The cash flow model models the underlying product being replicated. From the state variables generated in the scenarios, the cash flow model's purpose is to come up with a cash flow pattern for the product under consideration. For the purpose of this article, the cash flow model will be a cash flow model for mortgages that takes into account prepayment rates with behaviour as shown in figure 3.

> **a replicating portfolio can be used to specifically replicate mortgage prepayment risk**

The replicating instruments define the instruments that are allowed to be part of the replicating portfolio. It should also be possible to generate cash flows for the instruments under the scenario. The allowed instruments should fit the purpose of the goal of the replicating portfolio, e.g. for hedging purposes it makes sense to restrict oneself to tradable instruments, whereas for risk calculations one can also allow more exotic instruments to improve the quality of fit.

Finally, **the optimization problem** defines how a good a replicating portfolio is. There are many possibilities on to formulate the optimization problem. Some examples of choices to make are whether to optimize on the NPV of the cash flows or try to achieve some matching of NPV for certain time buckets, whether or not to include of sensitivities in the cost function to ensure matching of greeks, whether to add a trading cost or balance sheet constraint and under what norm to optimize. As a result of the optimization problem, a quality-of-fit report will be produced that shows how well a fit was achieved across various scenarios. Typically one observes that there's a trade-off between matching well on a path-by-path basis (high R2) versus having a good fit on sensitivities, which indicates a more robust fit.

Case study

To illustrate the preceding discussion, we'll now provide a case study on a fictive mortgage portfolio. We'll first describe each component of the replicating portfolio methodology as used in this case study and afterwards discuss the results.

Mortgage portfolio
For this case study, we'll work with a hypothetical mortgage portfolio of 1bln. This portfolio consists of 1,000 mortgages each with a notional of 1,000,000. Although obviously unrealistically large, we can think of these mortgages as being the result of some stratification process. With respect to the other mortgage characteristics, we've generated them through the simulation of a hypothetical evolution of interest rates and taking into account a shift over time to annuity mortgages. Figure 5 shows some characteristics off the portfolio generated.

From the figure one can observe that the annuity portfolios have longer maturities (as they became more popular later in time) and have also a bit higher time until their rate resets (time to reprice). The idea here is that as rates become lower, people in our simulated portfolio were interested in longer repricing periods. In line with the observations just described, figure 5b also shows the the various coupons per time-to-reprice bucket. From the figure it can be observed that indeed the average coupon on the annuity mortgages are lower (as our simulated mortgage rates decreased over time). Overall, the resulting mortgage portfolio is not unrealistic when compared with typical mortgage portfolios as a Dutch financial institution, although their duration is a bit higher.

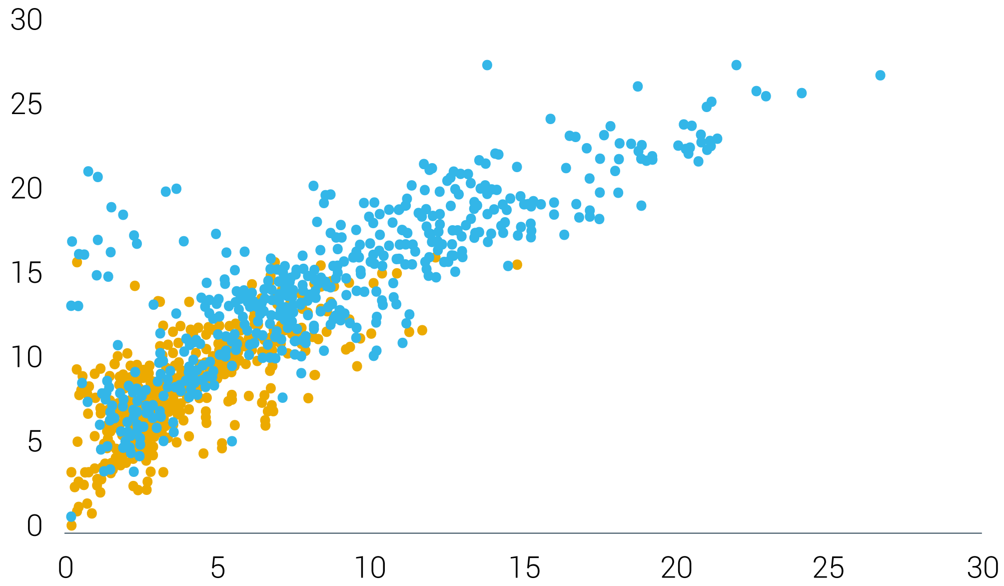

Figure 5a: distribution of time to repricing (x-axis) versus time to maturity (y-axis) for mortgage portfolio used

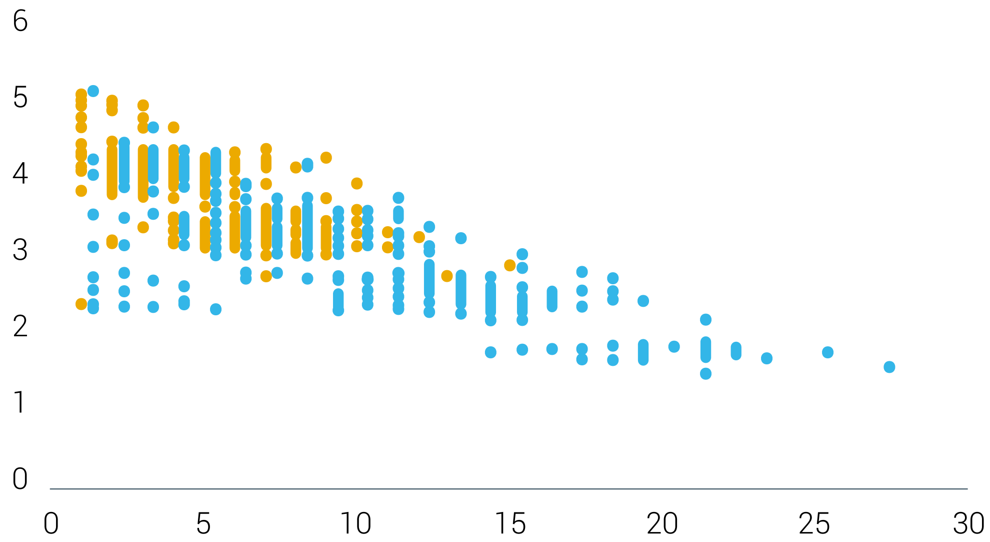

Figure 6b: distribution of coupon rates (y-axis) versus time to reprice for the mortgage portfolio used

Scenarios

To keep the case study small, we will consider three sets of scenarios that we'll simulate: a base scenario that has as a starting point a flat interest rate of 0.50%, a scenario that starts with a flat interest rate of 0.40% (a 10 basis point down shock) and a scenario that starts with a flat interest rate of 0% (a 50 basis point down shock). For each scenario, we'll generate 1,000 paths using a 2 factor hull white model. In addition, we'll assume that the add-on to generate mortgage rates from interest rates is constant at 200 basis points.

Cash flow model

To model prepayment behaviour, we'll use a simple ramp function. For any path S, at any point in time t, a mortgage m that reprices at T, we will define the prepayment incentive as $P_S^m(t,\ T)\ =\ r_m - r_S(t,T)$ where $r_S(t,T)$ is the mortgage rate under path S at time t for a mortgage that reprices at time T and r_m is the current mortgage rate for mortgage m. We then calculate the (annualized) prepayment rate as $CPR_S^m(t) = max\Big(0\%,\ min(6\%,\ P_S^m($. To avoid fitting the replicating portfolio just on the 'bulk' of the redemption profile of the mortgages, the cash flow model generates cash flows by subtracting from each time bucket the cash flow obtained from applying a 0% CPR to the portfolio. This way, really just the prepayment option is replicated and not the interest rate profile related to the discounting of the mortgage portfolio.

Allowed instruments

The instruments that we allow in the replication consist of zero coupon bonds as well as swaptions. We allow only long positions in the zero coupon bond and only short positions in the swaptions to replicate the risk profile. For all instruments the allowed maximum position equals EUR 2bln. For the zero coupon bonds, we allow tenors of 3m, 6m, 9m, ..., 30y. For the swaptions, we allow receiver swaptions with strikes from 1% to 5% and we allow all combinations underlying swap maturities of 1y, 3y, 5y, 10y, 15y and 20y, in combination of maturity dates of 2y, 3y, 4y, 5y, 10y, and 15y, but only those that the underlying cash flows do not extend beyond 30 years.

Optimization problem

For the optimization problem we've chosen to optimize only on the NPV of the entire cash flows and not for cash flows in various buckets. The problem was formulated as a quadratic problem and included a penalty term for greek mismatches and a fixed cost per instrument added to create more sparse results. Obviously, in a realistic application the scenarios used and the formulation of the optimization problem requires a great deal of thought and attention to make sure that the results are fit-for-use.

Results

The results of the simulation are shown in table 1 and figure 6. The figure shows a scatter plot of the NPV of the mortgage portfolio versus the NPV of the replicating portfolio, per path. Overlaid on the figure is the line y=x. In case of a perfectly replicating portfolio, all points would lie exactly on this line. As the figure indicates, a very good fit was obtained for this sample problem.

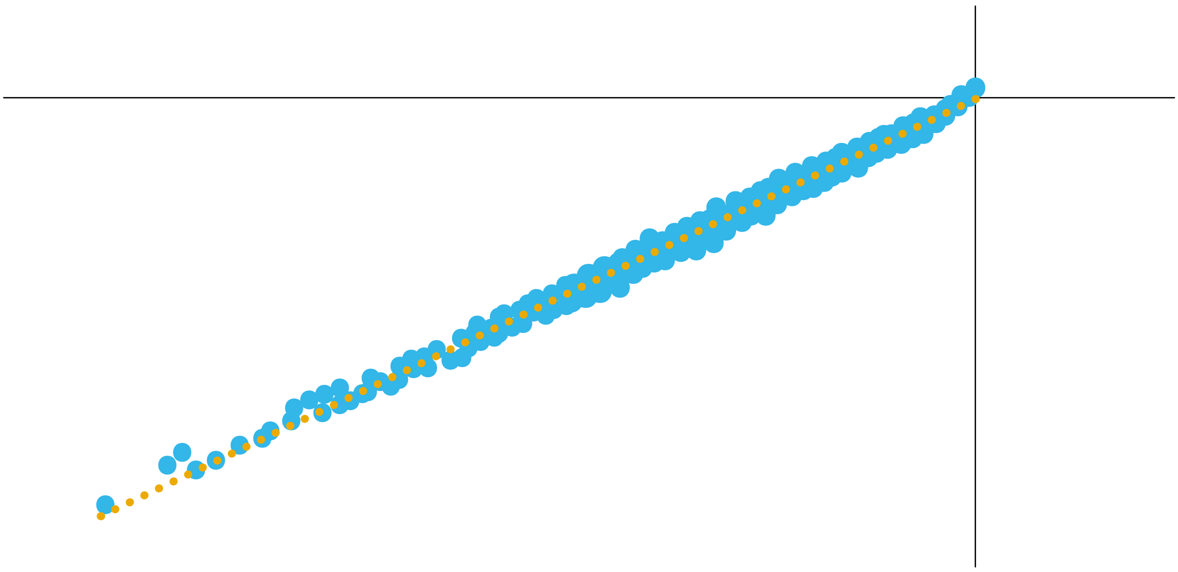

Figure 6: Mortgage NPV vs Replicating portfolio NPV per path

The good fit is also demonstrated by table 1, which shows some quality-of-fit statistics. In addition to the per-path fit looking good (as evidenced by e.g. the high R2), we also note that the change in value between the base scenario and the other scenarios are aligned. In particular the sensitivity towards a 10 basis point down shock shows that the delta of the replicating portfolio is close to that of the mortgage portfolio. The interest rate sensitivity found is consistent with a duration of ~10 years, which is a bit on the high side for current mortgage portfolios, but is consistent with a substantial part of our portfolio consisting of longer dated annuity mortgages and our prepayment model haven't been calibrated to match any specific durations on average (which in practical applications would be required). With current production rates, the figures provided are not an unreasonable proxy for how mortgage portfolios in financial institutions will look in a few years though.

	NPV		Abs Sensitivity				
Scenario	Product	RP	Product	RP	Sensitivity error	R^2	RMSE
Base	-11,650,836	-11,663,643				99.00%	782,120
-10bps	-12,728,659	-12,734,295	-1,077,822	-1,070,652	-0.70%	99.10%	806,186
-50bps	-17,571,965	-17,573,442	-5,921,129	-5,909,799	-0.20%	99.20%	923,102

Table 1: Quality-of-fit metrics for replicating portfolio

Another interesting insight from table 1 can be obtained by subtracting 5 times the sensitivity of the 10 basis point shock from sensitivity of the 50 bps down shock. This provides an indication of the instantaneous losses that would accrue due to optionality in case a perfect linear hedge was in place and a large shock occurred. Extrapolating that number to a 200 basis point shock as a proxy for a 1-in-200 shock for capital calculations, leads to a 2mm EUR loss on a reference portfolio of EUR 1bln, providing an indication of that this is quite a material risk. E.g. scaling up to the total mortgage market size as shown in figure AB, that would indicate EUR 1.5bln of losses for the entire market.

Summary and further directions

In this article, we've explained how that the prepayment option embedded in mortgages can be seen as an interest rate option. Valuation of this option is typically very quite time-consuming and one solution for efficient relevaluation for (risk) management purposes is using a replicating portfolio. For a hypothetical portfolio we've demonstrated that a replicating portfolio can be found to specifically match prepayment risk only. In addition, from the reports we've managed to produce some risk insights as well.

From a theoretical perspective, several topics can be further explored. For example, one can consider mortgage rates to consists of an interest rate component, a cost-of-funding component and a margin component. For the time being we've assumed the latter two to be static, but the analysis can be extended to model interest rates and cost-of-funding jointly. As another example, from a theoretical perspective, it can be further explored what instruments are expected to provide the best potential fit given the path-dependent notional of the prepayment option.

From a practical perspective, the scenario sets and allowed instruments in the replicating portfolio could be extended. More scenarios could be used to further match certain interest rate or volatility sensitivities, while the instrument set should be investigated for appropriateness. On top of that, for practical purposes the parameters for the optimization problem considered should be tweaked to provide a robust fit. Finally, for now we've considered only optimizing on NPVs. Often mortgage portfolios are held in an accrual environment and it's also desirable to find a replicating portfolio that doesn't only cover the risks on a mark-to-market basis, but also provides an (expected) matching accrual profile.

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1. To generate these numbers, a mortgage with a 30y term and a 15y refinancing period was assumed with a notional amount of 100, a fixed interest rate of 1% and a mortgage rate of 3%, causing the present value to be larger than 100.

2. Other alternatives exists, e.g. building tailor made distributed monte carlo systems or doing curve fitting

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Bram has a M.Sc. in Industrial Engineering and Management Science from the Eindhoven University of Technology. After graduation, he held various trading and management roles both in the Netherlands and abroad. In the little time left next to his job and family, he likes picking up new hobbies such as playing the piano (in which he is utterly awful).

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At Risk-at-Work we specialize in making models and providing solutions for quantitative risk management, trading and ALM topics. We pride ourselves on cutting through complexity by providing creative and practical solutions for hard problems. We do this by having a wealth of practical experience, state-of-the-art technical knowledge but first and foremost by understanding our clients and their problems and co-creating solutions with them that work.

What we do

Our client list covers all major Dutch financial institutions. We help our clients to get better, to go further. We help on topics where we believe we can offer unsurpassable quality. This means that we often work on topics that are on the intersection of quantitative finance, risk management and ALM. On these topics, we can help with providing insight, develop and implement models, review and improve models or just analyze understand data.

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